XIX. *Deſcription of a ſimple Micrometer for meaſuring ſmall Angles with the Teleſcope.* *By Mr.* Tiberius Cavallo, *F. R. S.*

Read June 2, 1791.

THE various teleſcopical micrometers, or machines which have been conſtructed for the meaſurement of ſmall angles, may be divided into two claſſes; namely, thoſe which have not, and thoſe which have, ſome movement amongſt their parts. The micrometers of the former ſort conſiſt moſtly of fine wires, or hairs, variouſly diſpoſed, and ſituated within the teleſcope, juſt where the image of the object is formed. In order to determine an angle with thoſe micrometers, a good deal of calculation is generally required. The micrometers of the other ſort, of which there is a great variety; ſome being made with moveable parallel wires, others with priſms, others again with a combination of lenſes, and ſo on; are more or leſs ſubject to ſeveral inconveniences, the principal of which are the following. 1ſt, Their motions generally depend upon the action of a ſcrew, and of courſe the imperfections of its threads, and the greater or leſs quantity of loſt motion, which is obſervable in moving a ſcrew, eſpecially when ſmall, occaſion a conſiderable error in the menſuration of angles. 2dly, Their complication and bulk renders them difficultly applicable to a variety of teleſcopes, eſpecially to the

pocket ones. 3dly, They do not meaſure the angle without ſome loſs of time, which is neceſſary to turn the ſcrew, or to move ſome other mechaniſm. 4thly, and laſtly, They are conſiderably expenſive, ſo that ſome of them coſt even more than a tolerably good teleſcope.

After having had long in view the conſtruction of a micrometer, which might be in part at leaſt, if not intirely, free from all thoſe objections; and, after various attempts, I at laſt ſucceeded with a ſimple contrivance, which, after repeated trials, has been found to anſwer the deſired end, not only from my own experience, but from that alſo of ſeveral friends, to whom it has been communicated.

This micrometer, in ſhort, conſiſts of a thin and narrow ſlip of mother of pearl finely divided, and ſituated in the focus of the eye-glaſs of a teleſcope, juſt where the image of the object is formed. It is immaterial whether the teleſcope be a refractor or a reflector, provided the eye-glaſs be a convex lens, and not a concave one, as in the Galilean conſtruction.

The ſimpleſt way of fixing it is to ſtick it upon the diaphragm, which generally ſtands within the tube, and in the focus of the eye-glaſs. When thus fixed, if you look through the eye-glaſs, the diviſions of the micrometrical ſcale will appear very diſtinct, unleſs the diaphragm is not exactly in the focus; in which caſe the micrometrical ſcale muſt be placed exactly in the focus of the eye-glaſs, either by puſhing the diaphragm backwards or forwards, when that is practicable; or elſe the ſcale may be eaſily removed from one or the other ſurface of the diaphragm by the interpoſition of a circular piece of paper or card, or by a bit of wax. This conſtruction is fully ſufficient when the teleſcope is always to be uſed by the ſame perſon; but when different perſons are to uſe it, then

the diaphragm, which ſupports the micrometer, muſt be conſtructed ſo as to be eaſily moved backwards or forwards, though that motion needs not be greater than about a tenth or an eighth of an inch. This is neceſſary, becauſe the diſtance of the focus of the ſame lens appears different to the eyes of different perſons, and therefore, whoever is going to uſe the teleſcope for the menſuration of any angle, muſt firſt of all unſcrew the tube, which contains the eye-glaſs and micrometer, from the reſt of the teleſcope, and, looking through the eye-glaſs, muſt place the micrometer where the diviſions of it may appear quite diſtinct to his eye.

In caſe that any perſon ſhould not like to ſee always the micrometer in the field of the teleſcope, then the micrometrical ſcale, inſtead of being fixed to the diaphragm, may be fitted to a circular perforated plate of braſs, wood, or even paper, which may be occaſionally placed upon the ſaid diaphragm.

I have made ſeveral experiments to determine the moſt uſeful ſubſtance for this micrometer. Glaſs, which I had ſucceſsfully applied for a ſimilar purpoſe to the compound microſcope, ſeemed at firſt to be the moſt promiſing; but it was at laſt rejected after ſeveral trials: for the diviſions upon it generally are either too fine to be perceived, or too rough; and though with proper care and attention the diviſions may be proportioned to the ſight, yet the thickneſs of the glaſs itſelf obſtructs in ſome meaſure the diſtinct view of the object. Ivory, horn, and wood, were found uſeleſs for the conſtruction of this micrometer, on account of their bending, ſwelling, and contracting very eaſily; whereas mother of pearl is a very ſteady ſubſtance, the diviſions upon it may be marked very

very eaſily, and, when it is made as thin as common writing paper, it has a very uſeful degree of tranſparency.

Tab. VIII. fig. 1. exhibits this micrometer ſcale, but ſhews it four times larger than the real ſize of one, which I have adapted to a three-feet achromatic teleſcope, that magnifies about 84 times. It is ſomething leſs than the 24th part of an inch broad; its thickneſs is equal to that of common writing paper; and the length of it is determined by the aperture of the diaphragm, which limits the field of the teleſcope. The diviſions upon it are the 200dths of an inch, which reach from one edge of the ſcale to about the middle of it, excepting every fifth and tenth diviſion, which are longer. The divided edge of it paſſes through the center of the field of view, though this is not a neceſſary precaution in the conſtruction of this micrometer. Two diviſions of the above deſcribed ſcale in my teleſcope are very nearly equal to one minute; and as a quarter of one of thoſe diviſions may be very well diſtinguiſhed by eſtimation, therefore an angle of one-eighth part of a minute, or of $7''\frac{1}{2}$, may be meaſured with it.

When a teleſcope magnifies more, the diviſions of the micrometer muſt be more minute; and I find, that when the focus of the eye-glaſs of the teleſcope is ſhorter than half an inch, the micrometer may be divided with the 500dths of an inch; by means of which, and the teleſcope magnifying about 200 times, one may eaſily and accurately meaſure an angle ſmaller than half a ſecond.

On the other hand, when the teleſcope does not magnify above 30 times, the diviſions need not be ſo minute: for inſtance, in one of DOLLOND's pocket teleſcopes, which when drawn out for uſe, is about 14 inches long, a micrometer with the

the hundredths of an inch is quite ſufficient, and one of its diviſions is equal to little leſs than three minutes, ſo that an angle of a minute may be meaſured by it.

In looking through a teleſcope, furniſhed with ſuch a micrometer, the field of view appears divided by the micrometer ſcale, the breadth of which occupies about one-ſeventh part of the aperture, and as the ſcale is ſemitranſparent, that part of the object, which happens to be behind it, may be diſcerned ſufficiently well to aſcertain the diviſion, and even the quarter of a diviſion, with which its borders coincide. Fig. 2. ſhews the appearance of the field of my teleſcope with the micrometer, when directed to the title page of the Philoſophical Tranſactions, wherein one may obſerve that the thickneſs of the letter C is equal to three-fourths of a diviſion, the diameter of the O is equal to three diviſions, and ſo on.

At firſt view one is apt to imagine that it is difficult to count the diviſions which may happen to cover or to meaſure an object; but upon trial it will be found, that this is readily performed; and even people, who have never been uſed to obſerve with the teleſcope, ſoon learn to meaſure very quickly and accurately with this micrometer; for, ſince every fifth and tenth diviſion is longer than the reſt, one ſoon acquires the habit of ſaying five, ten, fifteen, and then, by adding the other diviſions leſs than five, completes the reckoning. Even with a teleſcope, which has no ſtand, if the object end of it be reſted againſt a ſteady place, and the other end be held by the hand near the eye of the obſerver, an object may be meaſured with accuracy ſufficient for ſeveral purpoſes, as for the eſtimation of ſmall diſtances, for determining the height of a houſe, &c.

After

After having conſtructed and adapted this micrometer to the teleſcope, it is then neceſſary to aſcertain the value of the diviſions. It is hardly neceſſary to mention in this place, that though thoſe diviſions meaſure the chords of the angles, and not the angles or arches themſelves, and the chords are not as the arches, yet it has been ſhewn by all the trigonometrical writers, that in ſmall angles the chords, arches, ſines, and tangents, follow the ſame proportion ſo very nearly, that the very minute difference may be ſafely neglected: ſo that if one diviſion of this micrometer is equal to one minute, we may ſafely conclude, that two diviſions are equal to two minutes, three diviſions to three minutes, and ſo on. There are various methods of aſcertaining the value of the diviſions of ſuch a micrometer, they being the very ſame that are uſed for aſcertaining the value of the diviſions in other micrometers. Such are the paſſage of an equatorial ſtar over a certain number of diviſions in a certain time, the meaſuring of the diameter of the ſun, by computation from the focal diſtance of the object, and other lenſes of the teleſcope, the laſt of which, however, is ſubject to ſeveral inaccuracies; but as they are well known to aſtronomical perſons, and have been deſcribed in many books, need not be farther noticed in this Paper. However, for the ſake of workmen and other perſons not converſant in aſtronomy, I ſhall deſcribe an eaſy and accurate method of aſcertaining the value of the diviſions of the micrometer.

Mark upon a wall, or other place, the length of ſix inches, which may be done by making two dots or lines ſix inches aſunder, or by fixing a ſix-inch ruler upon a ſtand; then place the teleſcope before it ſo that the ruler or ſix-inch length may be at right angles with the direction of the teleſcope, and juſt 57 feet $3\frac{1}{2}$ inches diſtant from the object-glaſs of the teleſcope: this done, look through the teleſcope at the ruler or other

extenſion

extenſion of ſix inches, and obſerve how many diviſions of the micrometer are equal to it, and that ſame number of diviſions is equal to half a degree, or 30′; and this is all that needs be done for the required determination; the reaſon of which is, becauſe an extenſion of ſix inches ſubtends an angle of 30′ at the diſtance of 57 feet $3\frac{1}{2}$ inches, as may be eaſily calculated by the rules of plane trigonometry.

In one of DOLLOND's 14-inch pocket teleſcopes, if the diviſions of the micrometer be the hundredths of an inch, $11\frac{1}{2}$ of thoſe diviſions will be found equal to 30′, or 23 to a degree.

When this value has been once aſcertained, any other angle meaſured by any other number of diviſions is determined by the rule of three. Thus, ſuppoſe that the diameter of the ſun, ſeen through the ſame teleſcope, be found equal to 12 diviſions, ſay as $11\frac{1}{2}$ diviſions are to 30 minutes, ſo are 12 diviſions to $\left(\frac{12' \times 30'}{11,5}\right)$ 31′,3, which is the required diameter of the ſun.

Notwithſtanding the facility of this calculation, a ſcale may be made anſwering to the diviſions of a micrometer, which will ſhew the angle correſponding to any number of diviſions to mere inſpection. Thus, for the above-mentioned ſmall teleſcope the ſcale is repreſented in fig. 3. AB is a line drawn at pleaſure; it is then divided into 23 equal parts, and thoſe diviſions, which repreſent the diviſions of the micrometer that are equal to one degree, are marked on one ſide of it. The line then is divided again into 60 equal parts, which are marked on the other ſide of it; and theſe diviſions repreſent the minutes which correſpond to the diviſions of the micrometer: thus the figure ſhews, that ſix diviſions of the micrometer are equal to $15\frac{1}{2}$ minutes, $11\frac{1}{4}$ diviſions are nearly equal to 29

minutes, &c. What has been ſaid of minutes may be ſaid of ſeconds alſo, when the ſcale is to be applied to a large teleſcope.

Thus far this micrometer, and its general uſe, have been ſufficiently deſcribed, and mathematical perſons may eaſily apply it to the various purpoſes to which micrometers have been found ſubſervient. But as the ſimplicity, cheapneſs, and at the ſame time the accuracy of this contrivance, may render the uſe of it much more general than that of any other micrometer; and I may venture to ſay, that it will be found very uſeful in the army, and amongſt ſea-faring people, for the determination of diſtances, heights, &c.; I ſhall therefore ſubjoin ſome practical rules to render this micrometer uſeful to perſons unacquainted with trigonometry and the uſe of logarithms.

Problem I. The angle, not exceeding one degree, which is ſubtended by an extenſion of one foot being given, to find its diſtance from the place of obſervation.

N. B. This extenſion of one foot, or any other which may be mentioned hereafter, muſt be perpendicular to the direction of the teleſcope through which it is obſerved. The diſtances are reckoned from the object-glaſs of the teleſcope, and the anſwers obtained by the rules of this problem, though not exactly true, are however ſo little different from the truth, that the difference ſeldom amounts to more than two or three inches, which may be ſafely neglected.

Rule 1. If the angle be expreſſed in minutes, ſay, as the given angle is to 60, ſo is 687,55 to a fourth proportional, which gives the anſwer in inches.

2. If the angle be expreſſed in ſeconds, ſay, as the given angle is to 3600, ſo is 687,55 to a fourth proportional, which expreſſes the anſwer in inches.

3. If the angle be expreſſed in minutes and ſeconds, turn it all into ſeconds, and proceed as above.

Example. At what diſtance is a globe of one foot in diameter when it ſubtends an angle of two ſeconds?

$2 : 3600 :: 687{,}55 : \frac{3600 \times 687{,}55}{2} = 1237590$ inches, or $103132\frac{1}{2}$ feet, which is the anſwer required.

This calculation may be ſhortened; for ſince two of the three proportionals are fixed, their product in the firſt caſe is 41253, and in the other two caſes is 2475180; ſo that in the firſt caſe, *viz.* when the angle is expreſſed in minutes, you need only divide 41253 by the given angle; and in the other two caſes, *viz.* when the angle is expreſſed in ſeconds, divide 2475180 by the given angle, and the quotient in either caſe is the anſwer in inches.

Problem II. The angle, not exceeding one degree, which is ſubtended by any known extenſion, being given, to find its diſtance from the place of obſervation.

Rule. Proceed as if the extenſion were of one foot by Problem I. and call the anſwer B; then, if the extenſion in queſtion be expreſſed in inches, ſay, as 12 inches are to that extenſion, ſo is B to a fourth proportional, which is the anſwer in inches; but if the extenſion in queſtion be expreſſed in feet, then you need only multiply it by B, and the product is the anſwer in inches.

Example. At what diſtance is a man, ſix feet high, when he appears to ſubtend an angle of $30''$.

By Problem I. if the man were one foot high, the diſtance would be 82506 inches; but as he is ſix feet high, therefore multiply 82506 by 6, and the product gives the required diſtance, which is 495036 inches, or 41253 feet.

For greater conveniency, eſpecially in travelling, or in ſuch circumſtances in which one has not the opportunity of making even the eaſy calculations required in thoſe problems, I have calculated the following two tables; the firſt of which ſhews the diſtance anſwering to any angle from one minute to one degree, which is ſubtended by an extenſion of one foot; and the ſecond table ſhews the diſtance anſwering to any angle from one minute to one degree, which is ſubtended by a man, the height of which has been called an extenſion of ſix feet; becauſe, at a mean, ſuch is the height of a man when dreſſed with hat and ſhoes on. Theſe tables may be tranſcribed on a card, and may be had always ready with a pocket teleſcope furniſhed with a micrometer. Their uſe is evidently to aſcertain diſtances without any calculation; and they are calculated only to minutes, becauſe with a pocket teleſcope and micrometer it is not poſſible to meaſure an angle more accurately than to a minute.

Thus, if one wants to meaſure the extenſion of a ſtreet, let a foot ruler be placed at the end of the ſtreet; meaſure the angular appearance of it, which ſuppoſe to be 36′, and in the table you will have the required diſtance againſt 36′, which is 95½ feet. Thus alſo a man, who appears to be 49′ high, is at the diſtance of 421 feet.

T. CAVALLO.

Wells-ſtreet,
May 26, 1791.

Angles

Angles ſubtended by an extenſion of one foot at different diſtances.

Angles.	Diſtances in feet.	Angles.	Diſtances in feet.
Min. 1	3437,7	Min. 31	110,9
2	1718,9	32	107,4
3	1145,9	33	104,2
4	859,4	34	101,1
5	687,5	35	98,2
6	572,9	36	95,5
7	491,1	37	92,9
8	429,7	38	90,4
9	382,0	39	88,1
10	343,7	40	85,9
11	312,5	41	83,8
12	286,5	42	81,8
13	264,4	43	79,9
14	245,5	44	78,1
15	229,2	45	76,4
16	214,8	46	74,7
17	202,2	47	73,1
18	191,0	48	71,6
19	180,9	49	70,1
20	171,8	50	68,7
21	162,7	51	67,4
22	156,2	52	66,1
23	149,4	53	64,8
24	143,2	54	63,6
25	137,5	55	62,5
26	132,2	56	61,4
27	127,3	57	60,3
28	122,7	58	59,2
29	118,5	59	58,2
30	114,6	60	57,3

Angles ſubtended by an extenſion of ſix feet at different diſtances.

Angles.	Diſtances in feet.	Angles.	Diſtances in feet.
Min. 1	20626,8	Min. 31	665,4
2	10313.	32	644,5
3	6875,4	33	625.
4	5156,5	34	606,6
5	4125,2	35	589,3
6	3437,7	36	572,9
7	2946,6	37	557,5
8	2578,2	38	542,8
9	2291,8	39	528,9
10	2062,6	40	515,6
11	1875,2	41	503,1
12	1718,8	42	491,1
13	1586,7	43	479,7
14	1473,3	44	468,8
15	1375.	45	458,4
16	1289,1	46	448,4
17	1213,3	47	438,9
18	1145,9	48	429,7
19	1085,6	49	421.
20	1031,4	50	412,5
21	982,2	51	404,4
22	937,6	52	396,7
23	896,8	53	389,2
24	859,4	54	381,9
25	825.	55	375.
26	793,3	56	368,3
27	763,9	57	361,9
28	736,6	58	355,6
29	711,3	59	349,6
30	687,5	60	343,7

Fig. 2.

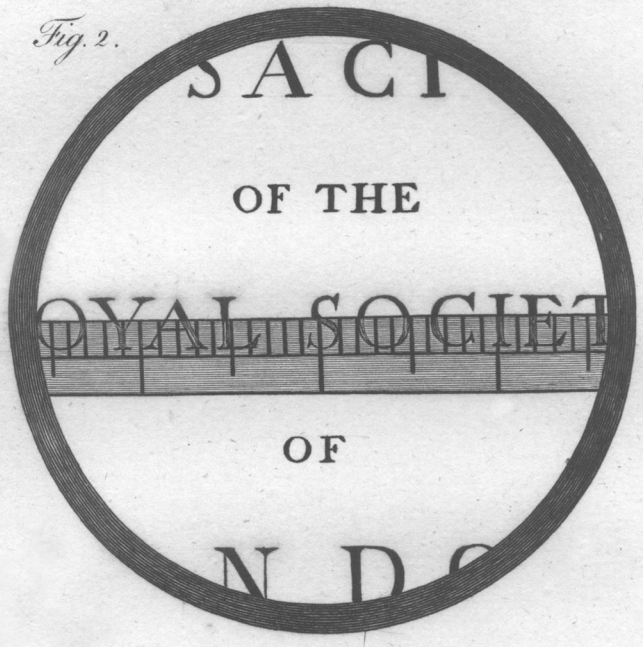

Fig. 1.

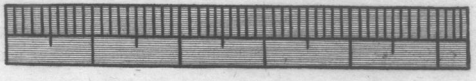

Fig. 3.

Cavallo del. *Basire Sc.*